Roll Number		

SET A

INDIAN SCHOOL MUSCAT
FIRST PRELIMINARY EXAMINATION
MATHEMATICS

CLASS: XII | Sub. Code: 041 | Time Allotted: 3 Hrs

10.01.2019 | Max. Marks: 100

General Instructions:

(i) All questions are compulsory.
(ii) Questions in section A are very short answer type questions carrying 1 mark each.
(iii) Questions in section B are short- answer type questions carrying 2 marks each.
(iv) Questions in section C are long answer I type questions carrying 4 marks each.
(v) Questions in section D are long answer II type questions carrying 6 marks each.

SECTION- A (Questions 1 to 4 carry 1 mark each)

1. If $\frac{1}{2}\cos^{-1}\left(\frac{4}{5}\right) = \tan^{-1} x$, find the value of x.

2. The equation of a line is given by $\frac{4-x}{2} = \frac{y+3}{3} = \frac{z+2}{6}$. Find the direction cosines of a line parallel to the above line.

3. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

4. A coin is tossed three times. Find $P(E|F)$ if, E = at least two heads; F = at most two heads.

OR

A card is drawn from a well- shuffled deck of 52 cards. If event E is that the card drawn is a spade and event F is that the card drawn is an ace. Show that the two events are independent.

SECTION- B (Questions 5 to 12 carry 2 marks each)

5. Form the differential equation of the family of circles touching the Y- axis at origin.

6. Find the value of λ, so that the following two lines are perpendicular:
$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$.

7. Find the slopes of the tangent and the normal to the curve $y = \frac{(x-2)(x+1)}{(x+3)}$ at the points where it cuts the X-axis.

OR

Prove that the function $y = \left[\frac{4\, sinx}{2+\cos\, x} - x\right]$ is an increasing function of x in $\left(0, \frac{\pi}{2}\right)$.

8. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

9. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

10. Show that the vectors $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$, $\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ are coplanar.

11. Find the equation of the plane that contains the point (-1, 3, 2) and perpendicular to each of the planes $x + 2y - 3z = 5$ and $3x + 3y - z = 0$.

OR

Find the equation of the plane passing through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and through the point (2, 2, 1).

12. Find the mean and variance of the number of heads in three tosses of a fair coin.

<u>SECTION- C (Questions 13 to 23 carry 4 marks each)</u>

13. Show that: $tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}cos^{-1}x^2$

14. If $\vec{a} = \hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 18$.

15. Using properties of determinants, prove that:

$$\begin{vmatrix} a + bx^2 & c + dx^2 & p + qx^2 \\ ax^2 + b & cx^2 + d & px^2 + q \\ u & v & w \end{vmatrix} = (x^4 - 1)\begin{vmatrix} b & d & q \\ a & c & p \\ u & v & w \end{vmatrix}$$

OR

Using properties of determinants, solve for x: $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0.$

16. Find the particular solution of the differential equation given that $y = 1$ when $x = 1$: $(3xy + y^2)dx + (x^2 + xy)dy = 0.$

17. Evaluate : $\int \frac{3x^2+4x+5}{(x-1)(x+2)(x-3)}dx$

18. If $f(x) = \begin{cases} ax + b & if \quad x > 2 \\ 7x - 4 & if \quad x = 2 \\ 3ax - 2b & if \quad x < 2 \end{cases}$ is continuous at $x = 2$, find the values of a and b.

19. Solve the differential equation: $cos^2x\left(\frac{dy}{dx}\right) + y = tanx$.

OR

Solve the differential equation: $ye^y\ dx = (y^3 + 2xe^y)\ dy$, given that $y(0) = 1$.

20. Evaluate $\int_1^3 (3x^2 + 1)dx$ by the method of limit of sum.

21. In a group of 400 people, 160 are smokers and non-vegetarians, 100 are smokers and vegetarians and the remaining are non- smokers and vegetarians. The probabilities of getting a particular chest disease are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disease. What is the probability that the selected person is a smoker and non- vegetarian?

22. Differentiate $y = x^{tanx} + (tanx)^x$ and find $\frac{dy}{dx}$.

23. Evaluate: $\int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

OR

Evaluate using properties of definite integrals : $\int_0^{\pi} \frac{x}{a^2cos^2x + b^2sin^2x} dx$

<u>SECTION- D (Questions 24 to 29 carry 6 marks each)</u>

24. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Also find the point of intersection.

25. A binary operation $*$ is defined on the set X = R-{-1} by $x * y = x + y + xy,\ x, y \in$ X. Check whether $*$ is commutative and associative. Find the identity element and also find the inverse of each element of X.

OR

Let A = Q ×Q where Q is the set of rational numbers and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Then find:
(i) the identity element of $*$ in A.
(ii) invertible element of (a, b) and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

26. Show that the semi- vertical angle of a cone of maximum volume and given slant height is $\tan^{-1}(\sqrt{2})$.

27. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & -1 \\ -2 & 1 & 1 \end{bmatrix}$, find A^{-1} and use A^{-1} to solve the following system of equations:

$$2x + y + 3z = 9,\ x + 3y - z = 2, -2x + y + z = 7.$$

OR

Using elementary transformation, find the inverse of the matrix: $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

28. A manufacturing company makes two types of teaching aids A and B of Mathematics for Class XII. Each type of aid A requires 9 hours for fabricating and 1 hour for finishing. Each type of aid B requires 12 hours for fabricating and 3 hours for finishing. For fabricating and finishing, the maximum hours available per week are 180 and 30 respectively. The company makes a profit of Rs.80 on each piece of type A and Rs.120 on each piece of type B. How many pieces of type A and type B should be manufactured per week to get a maximum profit? Make it as an LPP and solve graphically. What is the maximum profit per week?

29. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$ and the curve $x^2 = y$.

OR

Using integration, find the area of the following region: $\left\{(x, y): |x - 1| \leq y \leq \sqrt{5 - x^2}\right\}$

End of the Question Paper